Wetting and Dewetting of Thin Viscoelastic Drops

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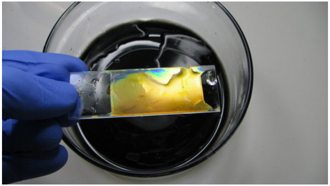
Outline

- Introduction
- Governing equations
- Numerical results:
  - Spreading and Receding Viscoelastic Drops vs Newtonian ones
  - Dynamic Contact Angle Analysis
- Conclusions

Introduction

Applications:

- Food Industry: ketchup, custard, starch suspensions.
- Chemical and Pharmaceutical Industries: toothpaste, shampoo.
- Coating processes in Material Sciences: glue.
- Biomedical Industry: blood, mucus, saliva.
- Green Energy materials: solar cells.



www.foodproductiondaily.com
MIT research

Governing Equations

Conservation laws:

ρ constant density $\Rightarrow \nabla \cdot \mathbf{u} = 0$

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla(p + \Pi) + \nabla \cdot \underline{\underline{\tau}} \ ,$$

where p is the hydrostatic pressure, Π is the pressure induced by body forces: van-der-Waals- type attraction/repulsion forces, $\underline{\underline{\tau}}$ the stress tensor. Jeffreys' model:

$$\underline{\underline{\tau}} + \lambda_1 \partial_t \underline{\underline{\tau}} = \eta(\underline{\underline{\dot{\gamma}}} + \lambda_2 \partial_t \underline{\underline{\dot{\gamma}}})$$

λ_1 relaxation time, λ_2 retardation time: $\lambda_2 = \lambda_1 \frac{\eta_s}{\eta_s + \eta_p} \Rightarrow \lambda_1 \geqslant \lambda_2$

η_s, η_p viscosity of Newtonian solvent and polymeric solute

Schematic

Thin-film approximation

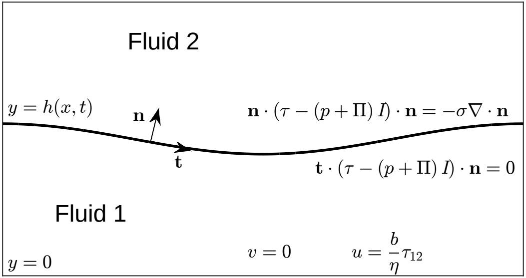


Figure: Schematic of the fluid interface and boundary conditions. Fluid 1 is the viscoelastic liquid, Fluid 2 an ambient (passive) gas.

Nondimensionalization

Scalings:

$$x = Lx^*\,,\ (y, h, h_\star, b) = H(y^*, h^*, h_\star^*, b^*)\,,\ (p, \Pi) = P(p^*, \Pi^*)\,,$$

$$u = Uu^*\,,\ v = \varepsilon U v^*\,,\ (t, \lambda_1, \lambda_2) = T(t^*, \lambda_1^*, \lambda_2^*)\,,\ \sigma = \frac{U\eta}{\varepsilon^3}\sigma^*\,,$$

$$\begin{pmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{pmatrix} = \frac{\eta}{T}\begin{pmatrix} \tau_{11}^* & \frac{\tau_{12}^*}{\varepsilon} \\ \frac{\tau_{21}^*}{\varepsilon} & \tau_{22}^* \end{pmatrix}\,,$$

where $H/L = \varepsilon \ll 1$ is the small parameter. Pressure is scaled with $P = \eta/(T\varepsilon^2)$, and time with $T = L/U$. We note that the Weissenberg number $Wi = \lambda_1 U/L = \lambda_1/T = \lambda_1^*$

Governing Equations

Long-wave approximation:

$$(1+\lambda_2\partial_t)h_t + \frac{\partial}{\partial x}\left\{(\lambda_2-\lambda_1)\left(\frac{h^2}{2}\mathbf{Q} - h\mathbf{R}\right)h_t + \left[(1+\lambda_1\partial_t)\frac{h^3}{3} + (1+\lambda_2\partial_t)bh^2\right]\frac{\partial}{\partial x}\left(\frac{\partial^2 h}{\partial x^2} + \Pi(h)\right)\right\} = 0\,,$$

$$\mathbf{Q} + \lambda_2\mathbf{Q}_t = -\frac{\partial}{\partial x}\left(\frac{\partial^2 h}{\partial x^2} + \Pi(h)\right),$$
$$\mathbf{R} + \lambda_2\mathbf{R}_t = -h\frac{\partial}{\partial x}\left(\frac{\partial^2 h}{\partial x^2} + \Pi(h)\right).$$

disjoining pressure: $\Pi(h) = \frac{\sigma(1-\cos\theta_e)}{Mh_\star}\left[\left(\frac{h_\star}{h}\right)^n - \left(\frac{h_\star}{h}\right)^m\right]$,
θ_e contact angle, $M = (n-m)/[(m-1)(n-1)]$, $(n > m > 1)$

Schematic: Circular Cap

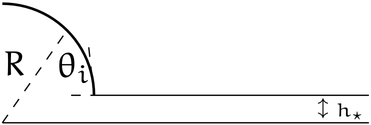
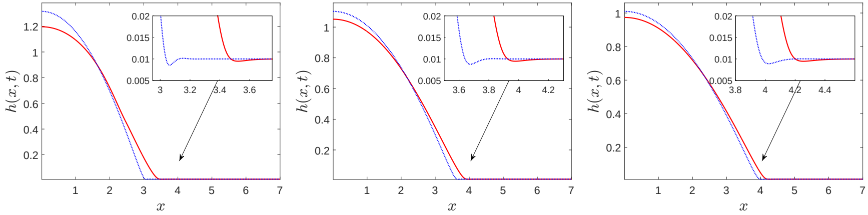


Figure: Schematic of the planar cap.

Spreading Drops: Comparison Movie

Viscoelastic drop $\lambda_1 = 15$, $\lambda_2 = 0.01$ (red solid curve) versus Newtonian drop with $\lambda_1 = \lambda_2 = 0$ (blue dotted curve).

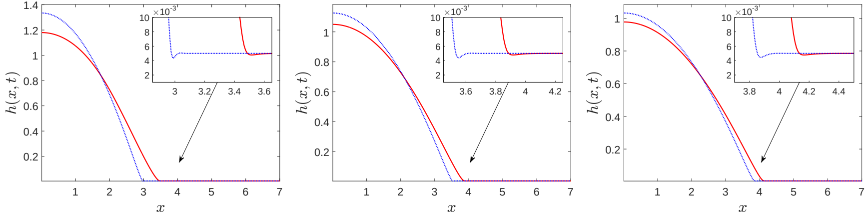
Spreading Drops: A close-up



(a) $t = 10$ (b) $t = 50$ (c) $t = 100$

Viscoelastic drop with $\lambda_1 = 15$, $\lambda_2 = 0.01$ (red solid curve) versus Newtonian drop with $\lambda_1 = \lambda_2 = 0$ (blue dotted curve). Precursor film thickness $h_\star = 0.01$

Spreading Drops: A close-up



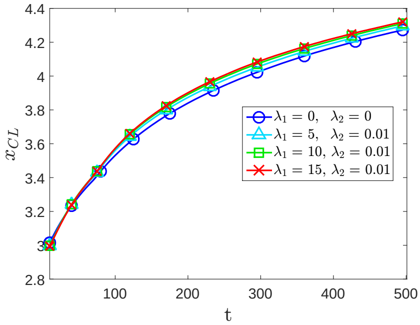
(a) $t = 10$ (b) $t = 50$ (c) $t = 100$

Viscoelastic drop with $\lambda_1 = 15$, $\lambda_2 = 0.01$ (red solid curve) versus Newtonian drop with $\lambda_1 = \lambda_2 = 0$ (blue dotted curve).

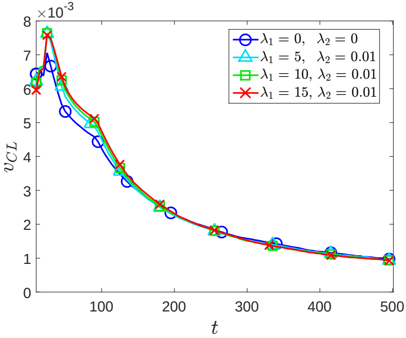
Precursor film thickness $h_\star = 0.005$

[M. A. Spaid, G. M. Homsy, Stability of Newtonian and viscoelastic dynamic contact lines, Phys. Fluids 8 (1996) 460–478.]

Spreading Drops - Contact Line Analysis



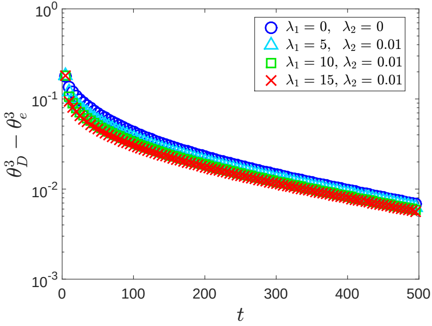
x_{CL} vs time



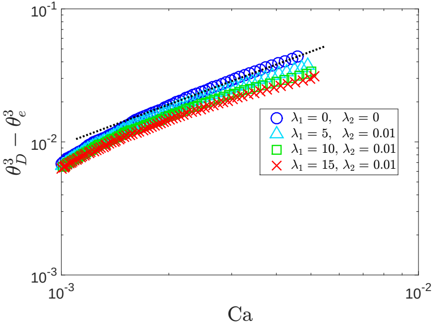
v_{CL} vs time

Spreading Drops - Dynamic C.A. Analysis

General Cox-Voinov law: $\theta_D^3 - \theta_e^3 \propto Ca^{\beta}$



$\theta_D^3 - \theta_e^3$ vs time in semi-log plot

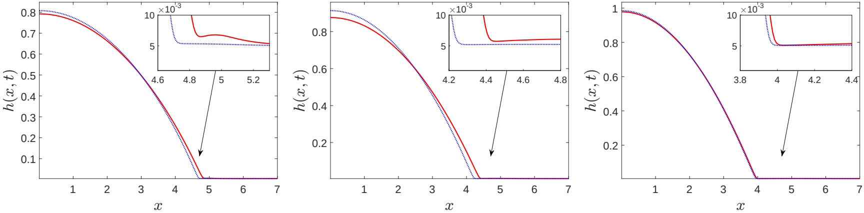


$\theta_D^3 - \theta_e^3$ vs Ca in log-log plot

Receding Drops: Comparison Movie

Viscoelastic drop $\lambda_1 = 15$, $\lambda_2 = 0.01$ (red solid curve) versus Newtonian drop with $\lambda_1 = \lambda_2 = 0$ (blue dotted curve).

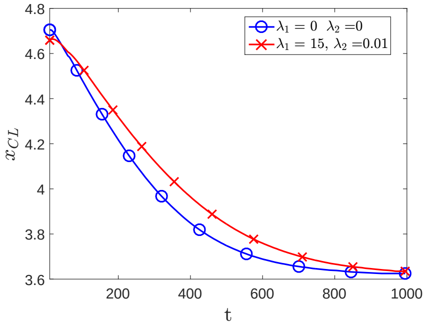
Receding Drops: A close-up



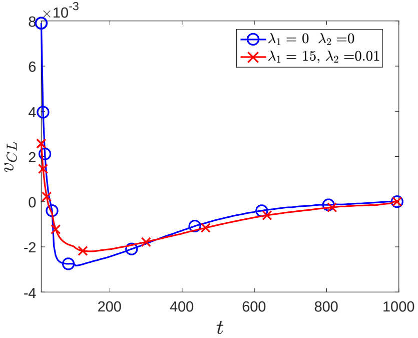
(a) $t = 100$ (b) $t = 350$ (c) $t = 1000$

Viscoelastic drop with $\lambda_1 = 15$, $\lambda_2 = 0.01$ (red solid curve) versus Newtonian drop with $\lambda_1 = \lambda_2 = 0$ (blue dotted curve). Precursor film thickness $h_\star = 0.005$

Receding Drops - Contact Line Analysis



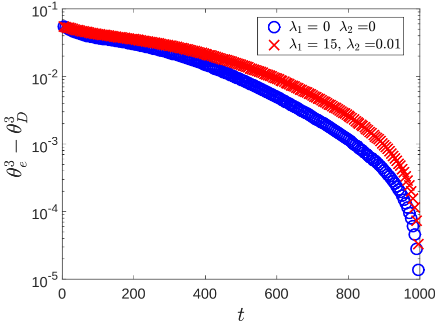
x_{CL} vs time



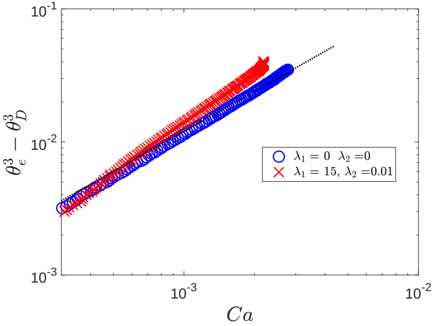
v_{CL} vs time

Receding Drops - Dynamic C.A. Analysis

General Cox-Voinov law: $\theta_D^3 - \theta_e^3 \propto Ca^\beta$



$\theta_D^3 - \theta_e^3$ vs time in semi-log plot



$\theta_D^3 - \theta_e^3$ vs Ca in log-log plot

Conclusions

- Viscoelasticity enhances spreading and slows down retraction
- Verified that viscoelastic effects more pronounced with thinner precursor thickness $h_\star$
- Cox-Voinox scaling law does not hold for viscoelastic fluids (lower for spreading, higher for receding)

Thank you!